

On Decreasing Returns to Bidders: Reserves, Regularity, and Risk Aversion

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ABSTRACT. When is the seller’s expected revenue from a second-price auction concave in the number of bidders? Watt (2025) shows this holds under monotone hazard rate (MHR) values, with reserves admitted only under a sample-size restriction. We give three extensions: arbitrary reserves under MHR; replacement of MHR by Myerson regularity, which admits all finite-mean Pareto distributions; and closure of regularity under increasing concave transformations, covering risk-averse sellers. The seller’s bidder-acquisition problem net of any convex utility cost is therefore concave under MHR for every reserve, and under regularity for every reserve at or above the (transformed) monopoly cutoff.

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1. Introduction

In an auction, the number of bidders is often implicitly a choice variable: attracting additional bidders raises competition but requires marketing, screening, and/or platform costs.

Watt (2025) proves that if the value distribution has a monotone hazard rate (“MHR”; see, e.g., Barlow and Proschan, 1975), then the expected k -th highest order statistic is nondecreasing and concave in the sample size. In a symmetric independent-private-values setting, Watt’s result delivers concavity of expected second-price revenue without a reserve and implies a well-behaved bidder-acquisition problem with convex costs. Watt (2025) also treats reserve prices, but the reserve result is stated under the additional sufficient condition $F(r) \leq 1 - 2/n$.

This note records three extensions of Watt’s work:

First, we show that the reserve restriction is not needed under MHR. For every fixed reserve $r \geq 0$ —including the revenue-maximizing reserve of Myerson (1981)—expected second-price revenue is nondecreasing and concave in the number of bidders. The proof follows Watt’s general outline, but keeps the reserve term rather than discarding it: that term supplies precisely the offset needed to control the positive part of the truncated second-order-statistic integral.

Second, we show that for reserve-free order statistics, the MHR assumption can be weakened. Let $Q := F^{-1}$, $h := f/\bar{F}$, and

$$\psi(u) := \frac{1}{h(Q(u))}, \quad \theta(u) := (1 - u)\psi(u).$$

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MHR is equivalent to ψ being nonincreasing; the concavity proof requires only the strictly weaker monotonicity of θ . In the smooth case this reduces to the pointwise inequality $-h'(x) \leq h(x)^2$, which is exactly the regularity condition of Myerson (1981): the virtual valuation $v(x) = x - 1/h(x)$ is nondecreasing. Theorem 3 below thus extends Watt's result from MHR to the full auction-theoretic regularity class—in particular, to all finite-mean Pareto distributions, none of which is MHR.

And third, we show that both MHR and Myerson regularity are closed under increasing concave transformations: if F is regular and φ is increasing concave, then the distribution of $\varphi(X)$ is regular. A risk-averse seller with concave utility φ therefore inherits the same concavity guarantees as under the baseline model.

The three results combine into a single payoff-concavity statement covering reserves, regularity, and risk aversion.

2. Setup and Preliminaries

For a sequence (a_n) , we write

$$\Delta a_n := a_n - a_{n-1}, \quad \Delta^2 a_n := a_{n+1} - 2a_n + a_{n-1}.$$

A sequence defined on $\{m, m+1, \dots\}$ is *concave* if $\Delta^2 a_n \leq 0$ for all $n \geq m+1$; likewise, it is *convex* if $\Delta^2 a_n \geq 0$ for all $n \geq m+1$.

Let X_1, \dots, X_n be i.i.d. draws from an absolutely continuous distribution F supported on an interval $J \subseteq [0, \infty)$ with finite lower endpoint and finite mean. Write f for the density and $\bar{F} := 1 - F$ for the survival function. The *hazard rate* of F is

$$h(x) := \frac{f(x)}{\bar{F}(x)}$$

at points with $\bar{F}(x) > 0$. We say that F has *monotone hazard rate (MHR)* if h is nondecreasing on the interior of its support J . We assume that f is continuous and positive on the interior of J , so that h is continuous and the pointwise quantities below $(h(r), \psi(F(r)))$ are unambiguously defined.

Let

$$X_{1:n} \leq \dots \leq X_{n:n}$$

denote the order statistics of X_1, \dots, X_n , and define $\mu_{k,n} := \mathbb{E}[X_{n-k+1:n}]$. Write $Q(u) := F^{-1}(u)$ for the quantile function. On $(0, 1)$,

$$Q'(u) = \frac{1}{f(Q(u))} = \frac{\psi(u)}{1-u}, \quad \psi(u) := \frac{1}{h(Q(u))}.$$

The MHR condition is equivalent to ψ being nonincreasing. The relaxed condition used in the sequel is

$$(\diamond) \quad \theta(u) := (1-u)\psi(u) \text{ is nonincreasing on } (0, 1).$$

MHR implies (\diamond) because both $1-u$ and $\psi(u)$ are nonincreasing.

When h is differentiable, (\diamond) is equivalent to $-h'(x) \leq h(x)^2$: a direct computation gives

$$\theta'(u) = (1-u)\psi'(u) - \psi(u) = -\frac{h(Q(u))^2 + h'(Q(u))}{h(Q(u))^3}.$$

Writing Myerson's virtual valuation in the form $v(x) := x - 1/h(x)$, we have $v'(x) = 1 + h'(x)/h(x)^2$, so the inequality is precisely $v'(x) \geq 0$. Condition (\diamond) is therefore the quantile-form expression of Myerson regularity, and remains meaningful when h is not differentiable. (Using $h'(x)/h(x) = (\log f)'(x) + h(x)$, the smooth form can also be written as $-(\log f)'(x) \leq 2h(x)$.)

We build on the following result of Watt (2025).

THEOREM 1 (Watt, 2025). *If F has MHR, then $n \mapsto \mu_{k,n}$ is nondecreasing and concave on $\{k, k+1, \dots\}$ for each $k \geq 1$.*

We interpret X_i as bidder i 's private value in a symmetric independent-private-values auction. In a second-price auction without a reserve and with $n \geq 2$, the seller's revenue is $X_{n-1:n}$. With a fixed reserve price $r \geq 0$, the object is sold only when at least one bidder's value reaches the reserve; the seller receives the second-highest value if at least two values reach the reserve, receives r if exactly one value reaches the reserve, and receives 0 otherwise. In the sequel we write the resulting random revenue as $\rho_n(r)$, with expectation $R_n(r)$. When bidder acquisition is costly, the seller chooses n to maximize expected auction revenue net of an acquisition cost $c(n)$.

3. Reserve prices under MHR

In the symmetric IPV model described in Section 2, for $n \geq 2$, second-price revenue with reserve $r \geq 0$ is

$$\rho_n(r) = X_{n-1:n} \mathbf{1}\{X_{n-1:n} \geq r\} + r \mathbf{1}\{X_{n-1:n} < r \leq X_{n:n}\}.$$

Let $R_n(r) := \mathbb{E}[\rho_n(r)]$ and $q := F(r)$. For $0 \leq q < 1$,

$$R_n(r) = r(1 - q^n) + \int_r^\infty [1 - F_{n-1:n}(x)] dx,$$

where $F_{n-1:n}(x) = nF(x)^{n-1} - (n-1)F(x)^n$.

THEOREM 2. *If F has MHR, then for every fixed reserve $r \geq 0$ the sequence $n \mapsto R_n(r)$ is nondecreasing and concave on $\{2, 3, \dots\}$.*

The proof of Theorem 2 uses a single-crossing inequality.

LEMMA 1. *Let $q \in [0, 1)$, and let K be integrable on $[q, 1]$. Suppose there is $s \in [q, 1]$ such that $K(u) \leq 0$ on $[q, s]$ and $K(u) \geq 0$ on $[s, 1]$. If p is nonnegative and nonincreasing on $[q, 1]$ with $p(q) = 1$, then*

$$\int_q^1 K(u)p(u) du \leq \max\left\{0, \int_q^1 K(u) du\right\}.$$

PROOF. Let $A(t) := \int_q^t K(u) du$. Because K crosses at most once, from negative to positive, A first weakly decreases and then weakly increases. Hence $\sup_{t \in [q, 1]} A(t) = \max\{A(q), A(1)\} = \max\{0, \int_q^1 K du\} =: M$. For absolutely continuous p , integration by parts gives

$$\int_q^1 K(u)p(u) du = A(1)p(1) + \int_q^1 A(u)(-p'(u)) du \leq M(p(1) + (p(q) - p(1))) = Mp(q) = M.$$

The general nonincreasing case follows by the Riemann–Stieltjes form of the same inequality, or by approximating p by smooth nonincreasing functions. \square

PROOF OF THEOREM 2. Monotonicity follows by coupling: the second-highest order statistic weakly increases when a draw is added, and so does $\rho_n(r)$ pointwise.

If $q = 1$, then $R_n(r) = 0$. If $q = 0$, the reserve never binds and $R_n(r) = \mu_{2,n}$, which is concave by Theorem 1. Hence assume $q \in (0, 1)$.

Change variables $u = F(x)$ and write $A_m(u) := 1 - mu^{m-1} + (m-1)u^m$. Then $R_m(r) = r(1 - q^m) + \int_q^1 A_m(u)\psi(u)/(1-u) du$. Direct expansion gives, for $n \geq 3$, $A_{n+1}(u) - 2A_n(u) + A_{n-1}(u) = (1-u)K_n(u)$, where

$$K_n(u) := u^{n-2}(1-u)(nu - (n-1)).$$

Hence

$$(1) \quad \Delta^2 R_n(r) = \int_q^1 K_n(u)\psi(u) du - r C_n(q), \quad C_n(q) := q^{n-1}(1-q)^2.$$

Reserve lower bound. Let $\ell := Q(0+) \geq 0$. Since ψ is nonincreasing under MHR,

$$(2) \quad r = \ell + \int_0^q \frac{\psi(u)}{1-u} du \geq \psi(q) \int_0^q \frac{du}{1-u} = \psi(q) \log \left(\frac{1}{1-q} \right).$$

Unweighted integral bound. Using $\frac{d}{du}[u^n(1-u)^2] = u^{n-1}(1-u)[n - (n+2)u]$, we obtain

$$(3) \quad q^n(1-q)^2 - \int_q^1 K_n(u) du = \int_q^1 (1-u)u^{n-2}P_n(u) du, \quad P_n(u) := (n+2)u^2 - 2nu + (n-1).$$

The discriminant of P_n is $4(2-n) \leq 0$ for $n \geq 2$ and the leading coefficient is positive, so $P_n \geq 0$ on $[0, 1]$ (with $P_2(u) = (2u-1)^2$). Hence the right side of (3) is nonnegative, and

$$(4) \quad \int_q^1 K_n(u) du \leq q^n(1-q)^2 = q C_n(q).$$

Combining. Set $p(u) := \psi(u)/\psi(q)$ on $[q, 1]$. Under MHR p is nonnegative and nonincreasing with $p(q) = 1$. The function K_n is negative on $[q, (n-1)/n)$ and positive on $((n-1)/n, 1]$, so Lemma 1 and (4) give

$$\int_q^1 K_n(u)p(u) du \leq \max \left\{ 0, \int_q^1 K_n(u) du \right\} \leq q C_n(q) \leq C_n(q) \log \left(\frac{1}{1-q} \right),$$

where the last step uses $q \leq -\log(1-q)$ for $q \in (0, 1)$. Dividing (1) by $\psi(q) > 0$ and using (2),

$$\frac{\Delta^2 R_n(r)}{\psi(q)} = \int_q^1 K_n(u)p(u) du - \frac{r}{\psi(q)} C_n(q) \leq 0. \quad \square$$

REMARK 1. Theorem 2 does not follow from Theorem 1 applied to the censored variable $\tilde{X} := \max(X, r)$. The truncated integral representation for $\Delta^2 \mathbb{E}[\tilde{X}_{n-1:n}]$ removes exactly the lower-tail region where Watt's argument obtains cancellation, so the remaining contribution need not be nonpositive. Smoothing the atom at r into a narrow density spike does not repair this either: the resulting distribution has hazard of order $1/\varepsilon$ on the spike and order $O(1)$ immediately above, violating MHR for every $\varepsilon > 0$. Our proof instead keeps the reserve term $r(1-q^n)$, and the lower bound (2) supplies the missing offset.

4. A relaxation of MHR without reserves

THEOREM 3. *Suppose F satisfies (\diamond) . Then for each $k \geq 1$ the sequence $n \mapsto \mu_{k,n}$ is nondecreasing and concave on $\{k, k+1, \dots\}$.*

PROOF. Monotonicity is unconditional: under the coupling that adds an extra draw, the k -th highest weakly increases.

For concavity, Watt's binomial recursion (Watt, 2025, Eq. (2)) gives

$$(5) \quad \Delta^2 \mu_{k,n} = \frac{\binom{n-1}{k-1}}{n-k+1} \int_J F^{n-k} \bar{F}^k [nF - (n-k+1)] dx.$$

For $k=1$ the bracket equals $n(F-1) \leq 0$, and concavity is immediate.

Now fix $k \geq 2$. Substituting $u = F(x)$, the sign of $\Delta^2 \mu_{k,n}$ is the sign of

$$I_{k,n} := \int_0^1 u^{n-k}(1-u)^{k-1} [nu - (n-k+1)] \psi(u) du.$$

With $\Phi(u) := u^{n-k+1}(1-u)^{k-1}$,

$$\Phi'(u) = u^{n-k}(1-u)^{k-2} [(n-k+1) - nu],$$

so the integrand of $I_{k,n}$ equals $-\theta(u)\Phi'(u)$ and

$$I_{k,n} = - \int_0^1 \theta(u)\Phi'(u) du.$$

The boundary terms in the integration by parts vanish. At $u = 1$, $\Phi(u) = O((1-u)^{k-1})$ for $k \geq 2$, and θ is bounded above by $\theta(\varepsilon)$ for any $\varepsilon \in (0, 1)$ by monotonicity. At $u = 0$, $\Phi(u) = O(u^{n-k+1})$ with $n-k+1 \geq 1$, and $u\theta(u) \rightarrow 0$: indeed, $\theta = (1-u)^2 Q' \leq Q'$, while Q is bounded near the lower endpoint, so θ is locally integrable near 0; for monotone θ , $(u/2)\theta(u) \leq \int_{u/2}^u \theta(t) dt \rightarrow 0$. Therefore, in the Stieltjes sense,

$$I_{k,n} = \int_0^1 \Phi(u) d\theta(u).$$

Under (\diamond) , $d\theta$ is a nonpositive signed measure and $\Phi \geq 0$, so $I_{k,n} \leq 0$ and $\Delta^2 \mu_{k,n} \leq 0$. \square

REMARK 2. Condition (\diamond) is strictly weaker than MHR. For a Pareto distribution with scale $x_m > 0$ and shape $\alpha > 0$, for example,

$$h(x) = \alpha/x, \quad -h'(x) = \alpha/x^2, \quad h(x)^2 = \alpha^2/x^2,$$

so h is strictly decreasing—never MHR—yet the smooth form of (\diamond) holds when $\alpha \geq 1$. The Pareto distribution has finite mean iff $\alpha > 1$; hence every finite-mean Pareto distribution is Myerson-regular and so satisfies (\diamond) .

5. Increasing concave transformations

Let φ be the seller's utility or any strictly increasing transformation. If $Z := \varphi(X)$, then $\varphi(X_{n-k+1:n}) = Z_{n-k+1:n}$, so concavity of $n \mapsto \mathbb{E}[\varphi(X_{n-k+1:n})]$ is an order-statistic question for Z . The next result shows that both MHR and Myerson regularity are closed under increasing concave transformations.

THEOREM 4. *Let $\varphi : [0, \infty) \rightarrow [0, \infty)$ be continuously differentiable, strictly increasing, and concave, with $\varphi(0) = 0$ and $\varphi' > 0$. Let $Z := \varphi(X)$.*

- (i) *If F has MHR, then Z has MHR.*
- (ii) *If F satisfies (\diamond) , then Z satisfies (\diamond) .*

Consequently, if F satisfies (\diamond) , then $n \mapsto \mathbb{E}[\varphi(X_{n-k+1:n})]$ is nondecreasing and concave on $\{k, k+1, \dots\}$ for each $k \geq 1$.

PROOF. Strictly increasing φ gives $Q_Z(s) = \varphi(Q(s))$ and $h_Z(Q_Z(s)) = h(Q(s))/\varphi'(Q(s))$. Hence

$$\psi_Z(s) := \frac{1}{h_Z(Q_Z(s))} = \varphi'(Q(s))\psi(s), \quad \theta_Z(s) = (1-s)\psi_Z(s) = \varphi'(Q(s))\theta(s).$$

Because Q is nondecreasing and φ' is nonincreasing, $s \mapsto \varphi'(Q(s))$ is nonnegative and nonincreasing.

(i) If F has MHR then ψ is nonincreasing. The product $\psi_Z = \varphi'(Q)\psi$ of two nonnegative nonincreasing functions is nonincreasing, so Z has MHR.

(ii) If F satisfies (\diamond) then θ is nonincreasing. The same product argument gives θ_Z nonincreasing, so Z satisfies (\diamond) . The final claim follows from Theorem 3 applied to Z . \square

6. Auction payoffs

Normalize the seller's utility for revenue so that $\varphi(0) = 0$. For $n \geq 2$, define

$$V_n(r) := \mathbb{E}[\varphi(\rho_n(r))], \quad Z := \varphi(X), \quad a := \varphi(r), \quad q := F(r).$$

Because φ is increasing,

$$V_n(r) = \mathbb{E}[\max\{a, Z_{n-1:n}\}] - a q^n,$$

which is exactly $R_n^Z(a)$, the risk-neutral reserve revenue for the distribution of Z .

THEOREM 5. *Let $\varphi : [0, \infty) \rightarrow [0, \infty)$ be continuously differentiable, strictly increasing, and concave, with $\varphi(0) = 0$ and $\varphi' > 0$. Let $r \geq 0$, and let $c : \{2, 3, \dots\} \rightarrow \mathbb{R}$ be a discretely convex bidder-acquisition cost, subtracted from the seller's payoff. Then $n \mapsto V_n(r) - c(n)$ is concave on $\{2, 3, \dots\}$ in either of the following cases:*

- (i) F has MHR; or
- (ii) F satisfies (\diamond) , and whenever $0 < F(r) < 1$ the reserve satisfies

$$(6) \quad \varphi(r) \geq \frac{\varphi'(r)}{h(r)}.$$

If, in addition, $c(n) - \mathbb{E}[\varphi(X_{n:n})] \rightarrow \infty$, then $V_n(r) - c(n)$ has a finite maximizer.

PROOF. By Theorem 4, if F has MHR then so does Z ; case (i) then follows from Theorem 2 applied to Z with reserve a .

For case (ii), treat boundary values of q first. If $F(r) = 0$, the reserve never binds and $V_n(r) = \mathbb{E}[Z_{n-1:n}]$, which is concave by Theorems 3 and 4. If $F(r) = 1$, then $V_n(r) = 0$. Hence, we assume $q \in (0, 1)$.

For $n \geq 3$, the calculation (1) applied to the distribution of Z gives

$$\Delta^2 V_n(r) = \int_q^1 K_n(s) \psi_Z(s) ds - a C_n(q).$$

Set $\Phi_n(s) := s^{n-1}(1-s)$ and $\Theta_Z(s) := (1-s)\psi_Z(s)$. Then $-\Phi'_n(s) = s^{n-2}(ns - (n-1))$ and $K_n(s)\psi_Z(s) = -\Theta_Z(s)\Phi'_n(s)$. By Theorem 4, Θ_Z is nonincreasing. Integrating by parts on $[q, 1 - \varepsilon]$ and then letting $\varepsilon \downarrow 0$ yields, in the Stieltjes sense,

$$\int_q^1 K_n(s) \psi_Z(s) ds = \Theta_Z(q)\Phi_n(q) + \int_q^1 \Phi_n(s) d\Theta_Z(s) \leq \Theta_Z(q)\Phi_n(q) = C_n(q)\psi_Z(q),$$

where the inequality uses the facts that $\Phi_n \geq 0$ and $d\Theta_Z \leq 0$. Since $\psi_Z(q) = \varphi'(r)\psi(q) = \varphi'(r)/h(r)$,

$$\Delta^2 V_n(r) \leq C_n(q) \left[\frac{\varphi'(r)}{h(r)} - \varphi(r) \right] \leq 0$$

by the hypothesis (6). Subtracting off a discretely convex c preserves concavity.

Finally, $\rho_n(r) \leq X_{n:n}$ and φ is increasing, so $V_n(r) \leq \mathbb{E}[\varphi(X_{n:n})]$; the growth condition forces $V_n(r) - c(n) \rightarrow -\infty$, and a finite maximizer exists. \square

REMARK 3. For risk-neutral utility $\varphi(x) = x$, (6) becomes $rh(r) \geq 1$, equivalently $v(r) := r - 1/h(r) \geq 0$: the reserve sits at or above Myerson's monopoly cutoff $r^* := \inf\{x \in J : v(x) \geq 0\}$. In transformed-value form, (6) is $v_Z(\varphi(r)) \geq 0$ for the virtual valuation of Z . For concave φ with $\varphi(0) = 0$, the tangent inequality $\varphi(r) \geq r\varphi'(r)$ implies that the risk-neutrality condition is sufficient for (6); risk aversion enlarges the admissible reserve set. Hence under regularity, case (ii) admits all reserves at or above Myerson's monopoly cutoff—the natural reference, since the Myerson-optimal reserve coincides with the cutoff. A concrete example where (\diamond) holds but MHR fails is

$$F(x) = \begin{cases} \frac{3}{5}x, & 0 \leq x \leq 1, \\ 1 - \frac{2}{5}x^{-3/2}, & x \geq 1, \end{cases} \quad h(x) = \begin{cases} \frac{3}{5-3x}, & 0 \leq x \leq 1, \\ \frac{3}{2x}, & x \geq 1. \end{cases}$$

The hazard rises on $[0, 1]$ and falls on $[1, \infty)$, so F is not MHR. It nevertheless satisfies (\diamond) : on $[0, 1]$, h is increasing, so $-h' \leq h^2$ trivially; on $[1, \infty)$, $-h'(x) = \frac{2}{3}h(x)^2$; and θ is continuous and nonincreasing at the join $x = 1$. For risk-neutral utility, $rh(r) = 3r/(5-3r)$ on $[0, 1]$, so (6) fails for $r < 5/6$ and holds for $r \geq 5/6$; the monopoly cutoff here is $r^* = 5/6$.

7. Discussion

Condition (\diamond) is the quantile-form expression of Myerson regularity (as we noted in Section 2). Within the class of distributions for which θ has bounded variation, the proof of Theorem 3 shows that concavity of $n \mapsto \mu_{k,n}$ for every $k \geq 2$ is equivalent to the family of moment inequalities

$$(\Delta) \quad \int_0^1 u^{n-k+1}(1-u)^{k-1} d\theta(u) \leq 0, \quad n \geq k+1, k \geq 2.$$

The inequalities (Δ) can hold even when θ is not globally nonincreasing — i.e., for distributions outside the regularity class—but the resulting condition lacks the auction-theoretic interpretation that makes (\diamond) useful.

Reserve prices are different. Theorem 2 relies on the additional fact that ψ is nonincreasing below the reserve, which delivers the lower bound $r \geq \psi(F(r)) \log[1/(1-F(r))]$; condition (\diamond) alone does not imply this property, which is why the relaxed reserve result in Theorem 5 requires the boundary condition $\varphi(r) \geq \varphi'(r)/h(r)$.

Together, the three extensions clarify that the settings in which bidder acquisition has decreasing returns to scale include (1) arbitrary fixed reserves under MHR, (2) the full Myerson-regular class in the reserve-free case, and (3) risk-averse evaluation of auction revenues.

Finally, we note that the algebra of Theorem 2 also covers the $n = 2$ second difference. With the convention $A_1 \equiv 0$, $R_1(r) = r(1 - F(r))$ (the posted-price revenue), and the identity $A_3 - 2A_2 + A_1 = (1-u)K_2(u)$ together with $P_2(u) = (2u-1)^2 \geq 0$ supply the same algebraic inputs used for $n \geq 3$. Likewise, in the transformed-payoff setting one may set $V_1(r) = \varphi(r)(1 - F(r))$. Theorems 2 and 5 therefore extend to $\{1, 2, 3, \dots\}$, the relevant range for the launch-margin decision of whether to attract a second bidder—i.e., whether to hold an auction at all.

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